Solutions to math questions from the sample practice exam I

48. Evaluate the following: $5.6294 + 24.6998$

(A) 3.03292 (B) 8.09938 (C) 29.3292 (D) 30.3292 (E) 30.9938

METHOD I: *Just do it! Align the decimal points and add.*

METHOD II: ***Estimate**, check the last digit, and eliminate bad candidates. Namely, a number more than 5.5 added to a number more than 24.5 will be a number more than 30, choices (A), (B), and (C) are out; the last digits are a result of adding 4+8 = 12, so our winner must end in 2. Thus, **(E)** is the correct answer.*

49. If the positive difference between 3.2 and 1.09 is multiplied by 0.47, the result is:

(A) 0.09917 (B) 0.9917 (C) 1.64 (D) 2.58 (E) 9.917

METHOD I: *Remember to add a "0" to 3.20 so you can line up the decimal places. Then subtract 3.20 - 1.09 = 2.11. Then multiply 2.11 x 1.09 = **0.9917 (B).***

METHOD II: *This problem lends itself to **estimating**. Once you have 3.20 - 1.09 = 2.11, observe that 2.11 is a little more than 2 and 0.47 is a little less than ½, so the product is very close to 1. The only choice that is very close to 1 is **(B)**.*

50. What is the value of $\frac{1}{2} \div \left(\frac{1}{2} \div \frac{1}{8}\right)$?

(A) 4 (B) 8 (C) 16 (D) $\frac{1}{4}$ (E) $\frac{1}{8}$

METHOD I: *Just do IT with **PEMDAS** in mind. Remember that dividing is the same multiplying by a reciprocal. Parentheses first, so* $\frac{1}{2} \div \frac{1}{8} = \frac{1}{2} \cdot \frac{8}{1} = 4$. *Finally,* $\frac{1}{2} \div 4 = \frac{1}{2} \div \frac{4}{1} = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$. *So, **(E)** is the correct answer.*

METHOD II: *Think of a pizza pie divided into 8 slices in total, since 8 is the common denominator of all fractions involved. Follow PEMDAS, so answer how many slices, or* $\frac{1}{8}$*-s, go into a half the pie? It is 4. A half divided into four equal parts must be an eighth of a whole. **(E)** is the correct answer.*

51. In lowest terms the product of $\frac{3}{4}, \frac{5}{12}$, and $\frac{8}{15}$ is:

(A) $\frac{1}{6}$ (B) $\frac{15}{90}$ (C) $\frac{120}{720}$ (D) $1\frac{7}{10}$ (E) 6

"Find the product" *means* ***"multiply."***

METHOD I: *First multiply all the numbers in the numerator and denominator and then simplify. We can multiply the numbers in any order to get the same result:*

$$\frac{3}{4}\cdot\frac{5}{12}\cdot\frac{8}{15}=\frac{3\cdot(5\cdot 8)}{12\cdot(4\cdot 15)}=\frac{3\cdot 40}{12\cdot 60}=\frac{120}{12\cdot 60}=\frac{10}{60}=\boxed{\frac{1}{6}}\quad(E)$$

METHOD II: *You can make this problem a little simpler by looking for common factors since any number divided by itself equals 1 (except 0/0 which is undefined).*

$$\frac{3}{4}\cdot\frac{5}{12}\cdot\frac{8}{15}=\frac{3}{2\cdot 2}\cdot\frac{5}{2\cdot 2\cdot 3}\cdot\frac{2\cdot 2\cdot 2}{3\cdot 5}=\frac{1}{2\cdot 3}=\boxed{\frac{1}{6}}\quad(E)$$

52. What is the value of the following expression?

$$\frac{\frac{7}{2}-\frac{1}{4}}{\frac{2}{3}-\frac{1}{4}}$$

(A) $\frac{5}{36}$ (B) $\frac{65}{48}$ (C) 3 (D) $\frac{21}{4}$ (E) $\frac{39}{5}$

METHOD I: *A fraction also represents division, so the above expression is the same as: $\left(\frac{7}{2}-\frac{1}{4}\right)\div\left(\frac{2}{3}-\frac{1}{4}\right)$. Fractions in action, let's find out a common denominator, simplify, and divide.*
$\left(\frac{7}{2}-\frac{1}{4}\right)\div\left(\frac{2}{3}-\frac{1}{4}\right)=\left(\frac{14}{4}-\frac{1}{4}\right)\div\left(\frac{8}{12}-\frac{3}{12}\right)=\frac{13}{4}\div\frac{5}{12}=\frac{13}{4}\cdot\frac{12}{5}=\frac{39}{5}$, ***(E)***.

METHOD II: *Multiplying by 1 does not change the value of an expression, so let's find a common denominator of all fractions involved, e.g. 12, and multiply the whole expression by 1 in the form of $\frac{12}{12}=\frac{\frac{12}{1}}{\frac{12}{1}}$ to simplify it.*
$\frac{\left(\frac{7}{2}-\frac{1}{4}\right)}{\left(\frac{2}{3}-\frac{1}{4}\right)}\cdot\frac{\frac{12}{1}}{\frac{12}{1}}=\frac{42-3}{8-3}=\frac{39}{5}$.

53. What is the 185th digit in the following pattern 12345678910111213141516….

(A) 0 (B) 5 (C) 7 (D) 8 (E) 9

You are definitely not going to write out all 185 digits, so what is going on?
*This long string of digits starts with 9 one-digit numbers and then is followed by the 90 two-digit numbers from 10 to 99. Those two-digit numbers use 180 digits. That tells us that the 188th and 189th digits are "99," so the 186th and 187the are "98," and then the 184th and 185th are "97." Thus the 185th digit is **7** which is choice (**C)**.*

54. Copy Cat Copies charges 18 cents for the first copy and 12 cents for each additional copy. What is the greatest number of copies you can get for $3.00?

(A) 16 (B) 17 (C) 23 (D) 24 (E) 25

*We may have to **estimate** and **round** in this problem. How many cents are in $3.00? There are 100 cents in $1.00, so there will be 300 cents in $3.00. Let's subtract 18 cents for the first copy and divide the rest by 12 cents to count the additional copies.*

300 cents - 18 cents = 282 cents and $282 \div 12 = 141 \div 6 = 47 \div 2 = 23.5.$

After making copy number 1, we would be able to make 23 additional copies. In total, we can make 24 copies for $3.00. ***(D)***

55. The scores of Tim's first four out of six math tests are 82%, 90%, 78%, and 100%. If Tim wants to have a test average of 90%, what must the average of Tim's next two tests be?

(A) 90% (B) 92% (C) 95% (D) 97% (E) 100%

METHOD I: *If the average of six tests is 90, the sum of the grades on those six tests is 6 x 90 = 540. The sum of the first 4 tests is:*
82 + 90 + 78 + 100 = 82 + 78 + 90 + 100 = 160 + 190 = 350.
Tim needs 540 - 350 = 190 points on the other two tests, for an average of 190/2 = ***95 ,*** *which is choice (**C**).*

METHOD II: *Relative to 90, Tim's scores are "down 8," 90 even, "down 12," and "up 10." Overall his grades are "down 10," so he needs to be "up 10" on the next 2 exams or "up 5" on each of the next two tests, which means* ***95*** *on each, choice (**C**).*

56. Susan begins counting backward from 1298 by 4's, saying one number every 5 seconds. At the same time, Jim begins counting forward from 171 by 3's, saying one number every 5 seconds. What number will they both say at the same time?

(A) 640 (B) 644 (C) 648 (D) 650 (E) 654

Given a lot of time we could write it all out. We have a few minutes to do this, so let's look at numbers. Susan counts backward by 4's starting at 1298. Is 1298 ***divisible*** *by 4? Let's try it. No, it is not; it is 2 away from 1300 which is divisible by 4. So, every number Susan says cannot be divisible by 4. Jim's number, however, 171 is divisible by 3, so every number Jim says has to be divisible by 3. We are told that there will be a number they will say at the same time. This number must be a* ***multiple*** *of 3 and it cannot be a multiple of 4.*

How do we know if a number is divisible by 3?
If all of its digits add up to a number divisible by 3, then the number must be divisible by 3.

1+7+1 = 9, so 171 is divisible by 3.

Eliminate answers that are NOT divisible by 3, (A), (B), (D) do not work. We are left with the choices (C) and (E), but (C) is divisible by 4, so it is impossible. Our answer is ***(E)****. You can check to see that 654 is 2 away from 656 which is a multiple of 4.*

57. Gabe is taller than Helen, and Helen is shorter than Iris. Iris is shorter than both Jack and Keiko. Which of the following statements is true?

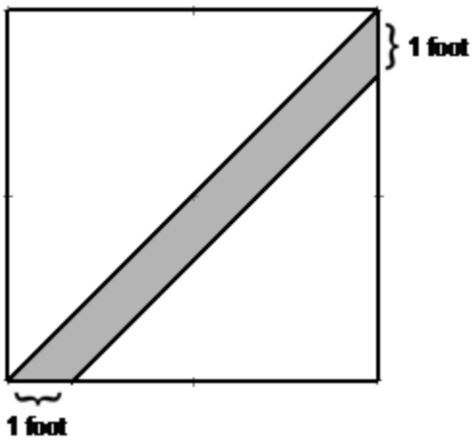
(A) The tallest person must be Gabe.
(B) The tallest person must be Jack.
(C) The tallest person must be Keiko.
(D) Either Jack or Keiko must be the tallest person.
(E) Gabe, Jack, or Keiko could be the tallest person.

Arrange the 5 names from shortest to tallest. You can just call them G, H, I, J, and K.

So far we have H G
H I
I J
I K

We do not know if G is before or after I and we do not know who comes first, J or K. The only valid conclusion we can make is that ***(E) Gabe, Jack, or Keiko could be the tallest person.***

58. It takes 24 feet of fence to surround a square piece of land in Linda's backyard. Linda wants to plant flowers everywhere but on the diagonal sidewalk (shaded in the diagram below). Note that there is 1 foot of sidewalk along two sides of the square as indicated in the diagram below. What is the area of the sidewalk?



(A) 5 ft^2 (B) 5.5 ft^2 (C) 6 ft^2 (D) 6.5 ft^2 (E) 7 ft^2

How many familiar figures are there in the diagram? A square, two right triangles, and the sidewalk in the shape of a trapezoid. How do areas of these figures relate?

The total area of the square =
area of the large right triangle + area of the small right triangle + area of the sidewalk.

So, we can compute the area of the sidewalk by subtracting areas of the two right triangles from the total area of the square. Let's do it. Since the square has 4 equal sides and its perimeter is 24ft, each side must be $24ft \div 4 = 6ft$ *long. So, the area of the square is* $6^2 = 36ft^2$. *The large right triangle is half of the square, so its area is half of the area of the square, namely it is* $36ft^2 \div 2 = 18ft^2$. *The smaller triangle has perpendicular sides that are 5ft long each, so its area is a half of the area with a side of 5ft, it must be* $\frac{1}{2} \cdot 5^2 ft^2 = \frac{25}{2} ft^2 = 12.5ft^2$. *To compute the area of the sidewalk, let's subtract.*

Area of the sidewalk = $36ft^2 - 18ft^2 - 12.5ft^2 = 5.5ft^2$; *the answer is (**B**).*

59. When the 69th even natural number is subtracted from the 119th odd natural number, what is the result? (The natural numbers are 1, 2, 3, 4,…)

(A) 49 (B) 51 (C) 99 (D) 101 (E) 103

The first even natural number is 2, the second is 4, the third is 6, so the 69th is 2 x 69 = 138. (Sneakier way: the 70th is 2 x 70 = 140, so the 69th is 138.)

The first odd natural number is 1, the second is 3, the third is 5. Observe that each is 1 less than the corresponding even number. Since the 119th even number is 2 x 119 = 238, the 119th odd number is 237.

Now let's answer the question: 237 - 138 = ***99****, which is choice (****C)****.*

60. What is the smallest prime number that can be written as the sum of three different prime numbers? (A prime number is a number that has only two factors, 1 and itself; 1 is not a prime number.)

(A) 10 (B) 15 (C) 17 (D) 19 (E) 23

We can use the answers and ***work backwards and eliminate some answer choices****. Since we are looking for a prime number, we need to exclude answers (A) and (B) as they are composite, e.g.* $10 = 2 \cdot 5$ *and* $15 = 3 \cdot 5$.

Let's explore the rest. The only even prime number is 2; all the others are odd. When we add three numbers and we want to have a result that is odd, all three of them must be odd numbers (the sum of two even numbers and one odd number is also odd, but there is only one even prime number!). So, 2 cannot be one of the three numbers we are adding.
Let's list odd prime numbers less than 23 and let's find a few sums:

3, 5, 7, 11, 13, 17, 19

We see that the smallest sum would be 3+5+7=15, but it is not prime, let's try another 3+5+11=19. All other sums would use larger numbers, so they would sum up to a larger value. Yay, we got our smallest prime number that is a sum of three different numbers. The answer is ***(D)****.*

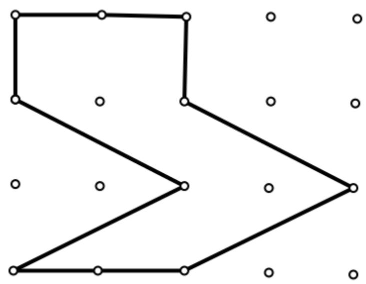
61. If you have six people in a room, and each person shakes hands with every other person exactly once, how many total handshakes happen?

(A) 6 (B) 12 (C) 15 (D) 21 (E) 720

METHOD I: *Each person shakes hands with 5 other people. That sounds like 6 x 5 = 30 handshakes. But, when I shake your hand, you shake mine and we both count that handshake. Thus each handshake is counted twice, so the total number of handshakes is 30/2 =* ***15****, which is choice (****C)****.*

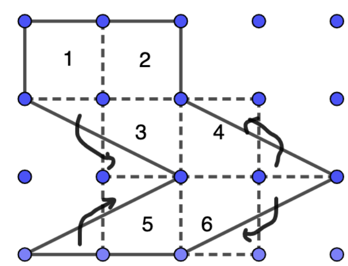
METHOD II: *Call the 6 people A, B, C, D, E, and F. A shakes 5 other hands. B already shook hands with A, so B shakes 4 other hands. C already shook hands with A and B, so C shakes 3 other hands. D already shook hands with A, B, and C, so D shakes 2 other hands. E has already shaken hands with A, B, C, and D, so E only shakes 1 other hand. F has already shaken hands with the other 5 people so there are no more handshakes. The total number of handshakes is 5 + 4 + 3 + 2 + 1 =* ***15****, which is choice (****C)****.*

62. In this grid, the dots are spaced one unit apart, horizontally and vertically. What is the number of square units enclosed by the solid line?



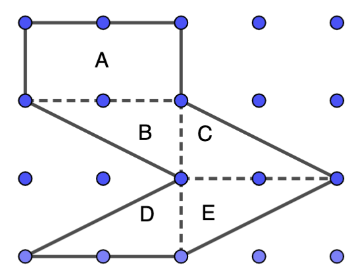
(A) 5.5 (B) 6 (C) 6.5 (D) 7 (E) 7.5

METHOD I: **Draw on the diagram and estimate**.
Parts of the diagram look like they fit!
Count the squares. The answer is **(B)**.

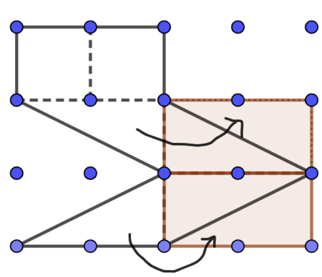


METHOD II : **Apply the formulas for the area of a rectangle (**$length \cdot width$**) and a triangle** $\left(\frac{1}{2} \cdot base \cdot height\right)$**.**

Our shape is made up of one rectangle whose area is $2 \cdot 1 = 2$ squared units and four identical triangles, each with area $\frac{1}{2} \cdot 2 \cdot 1 = 1$ squared unit. **The total area equals the sum of its parts**, so it equals $2 + 1 + 1 + 1 + 1 = 6$ squared units or **(B).**



METHOD III: Cut and paste!



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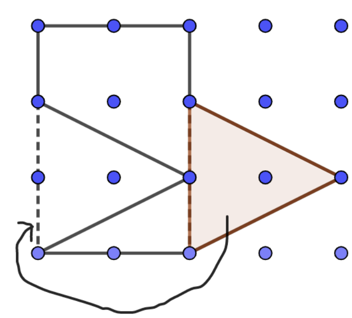


63. In the expression below, each letter represents a one-digit number. Where the same letter appears, it represents the same number in each case. Each distinct letter represents a different number. In order to make the equation true, what number must replace C?

$$\begin{array}{r} \text{AAA} \\ \text{AAB} \\ +\ \ \underline{\text{ABC}} \\ 2012 \end{array}$$

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

When you add 3 different digits, the sum is at least 0 + 1 + 2 = 3 and at most 7 + 8 + 9 = 24. That tells us that, in each column, the sum can lead us to carry 0, 1, or 2 to the column to its left.

Looking at the hundreds' column, 3 A's add up to 20; that can only happen if A = 6 and 2 was carried over from the 10's column.

Next, looking at the tens' column 6 + 6 + B + carry from the ones' column = 21 (since 2 must carry over to the hundreds' column). Therefore, B + carry from the ones' column = 21 - 12 = 9, so B = 7 or 8 or 9.

Finally, looking at the ones' column, 6 + B + C = 22 (since B is at least 7, 6 + B is at least 13, so the sum of 6 + B + C > 12, so the sum in the ones' column must be 22). Thus 2 will carry to the tens place, so B + 2 = 9 and B = 7. Finally 6 + 7 + C = 22, so C = 22 - 13 = ***9****.* ***(E)***

64. 15 people received an email and sent it to 3 different friends each, who in turn each sent it to 2 new people. What percent of the total number of people who received the e-mail are the original 15 people?

(A) 8% (B) 10% (C) 11% (D) $16\frac{2}{3}\%$ (E) 20%

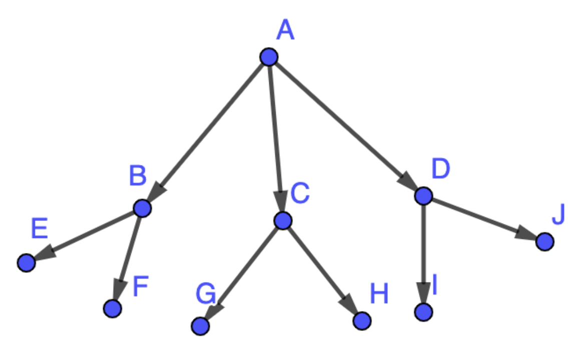
METHOD I: ***Bookkeeping is counting.*** *15 start, each sends the email to 3 people, so now* $15 \cdot 3 = 45$ *new people have the email. Since each one from the 45 new people sends it to 2 new, we now have* $45 \cdot 2 = 90$ *new people get the email. The total number of people who got the email is:*

15 (the first group) + 45 + 90 = 150

Our first group is $\frac{15}{150} = \frac{1}{10}$ *of the total which is equivalent to* $\frac{1}{10} \cdot 100\% = 10\%$. *The answer is* ***(B)***.

METHOD II: ***A picture is worth a thousand words!***

This picture represents a chain from one person from the first group. Observe that there are 10 people in total that got the email and the first person, A, is $\frac{1}{10}$ *of the total.*
The answer is ***(B)***.



65. Susan is traveling from Queens to Staten Island via Manhattan. There are 3 different trains that travel from her home in Queens to Times Square, 5 trains that run from Times Square to the Staten Island Ferry, and 2 buses that run from the Staten Island Ferry to her home in Staten Island. How many different ways can Susan get from Queens to her home by train and bus, passing through Times Square and the Staten Island Ferry?

(A) 3 (B) 10 (C) 13 (D) 15 (E) 30

Each of the 3 trains from Queens to Times Square can be followed by one of 5 trains from Times Square to the Staten Island Ferry, for a total of 3 x 5 = 15 possible routes from Queens to the ferry. Each of these 15 routes can be followed by one of the 2 buses from the ferry to her home, for a total of 15 x 2 = ***30*** *different ways.* ***(E)***

66. Sam sold his bike to Suzie for 35% more than he paid for it. Suzie then sold the bike to Stanley for 20% more than she paid for it. What percent of the original price did Stanley pay for the bike?

(A) 55% (B) 62% (C) 155% (D) 162% (E) 168.75%

We do not have enough information to figure out the first price of the bike, so let's say Sam paid $100 for it (100 is a nice number for percentages). Suzie paid 35% more than Sam, so 35% of $100 is $35, so Suzie paid $135 for the bike. Stanley paid 20% more than Suzie, so we need to compute 20% of $135 or twice 10% of $135 , which is $2 \cdot 10\% \cdot \$135 = 2 \cdot \$13.5 = \$27$, *so Stanley's prize was $135 + $27 = $162.*

$162 is 162% of $100, so ***(D)*** *is the answer.*

67. Ella spent her whole allowance of \$2.00, plus the 16¢ she had left over from last week, on bubble gum. If the pieces of gum had been a penny cheaper, she would have received three more pieces than she did. How many pieces did she actually buy?

(A) 8 (B) 9 (C) 16 (D) 24 (E) 27

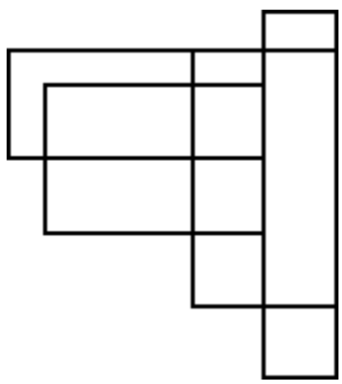
It is easier to work with cents in this problem.
Ella spent 216 cents. You can make a prime factor tree to see that:

$$216 = 2^3 \cdot 3^3 \ \ so$$

$$216 = 1 \cdot 216 = 2 \cdot 108 = 3 \cdot 72 = 4 \cdot 54 = 6 \cdot 36 = 8 \cdot 27 = 9 \cdot 24 = 12 \cdot 18$$

Looking at the list of possibilities*, you can see that at 9 cents each she could get 24 pieces of gum while at 8 cents each she would get 27 pieces, so she actually got* ***24*** *pieces.* ***(D)***

68. How many 4-sided figures are in this diagram?



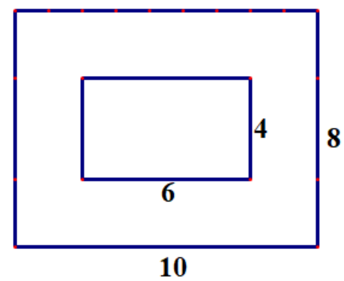
(A) 10 (B) 16 (C) 21 (D) 25 (E) 28

This is a problem for an artist! Four sided figures seem like rectangles. Some do not ***overlap*** *and some do, so we must count carefully and* ***organize****. For example, in the "strip" on the right there are 3 non-overlapping rectangles, but there are also 3 overlapping rectangles, so in total in just that "strip" there are 6 rectangles.*
The question is asking for four sided figures, so we need to count overlapping shapes also. *When carefully counted there should be 25.* ***(D)***

69. If a photo measuring 4 inches by 6 inches is placed in a photo frame that is 2 inches wide all around, what percent of the framed photo is the photo itself?

(A) 24% (B) 30% (C) 42% (D) 50% (E) 56%

Draw a picture, it helps!



Since the frame is 2 inches wide all around, the width of the framed photo is 4 + 2 + 2 = 8 inches (2 inches are added above and below the picture) and its width is 6 + 2 + 2 = 10 (2 inches are added on the left and on the right side).

The percent of the framed photo that is just the photo itself is:

$$\frac{Area\ of\ photo}{Area\ of\ photo + frame} = \frac{4 \cdot 6}{8 \cdot 10} = \frac{24}{80} = \frac{3}{10} = \boxed{30\%}$$

(B)

70. Steve, Jerry, and Ron were paid \$29.25 to remove garden gnomes. They each worked four hours, except for Ron, who was 45 minutes late. How much of the \$29.25 should Ron receive?

(A) \$2.60 (B) \$7.80 (C) \$8.45 (D) \$9.75 (E) \$10.40

Let's eliminate a few answer choices. If all three friends worked together for the same amount of time, each should get:

$\$29.25 \div 3 = (\$30 - \$0.75) \div 3 = \$10 - \$0.25 = \$9.75$ *each.*

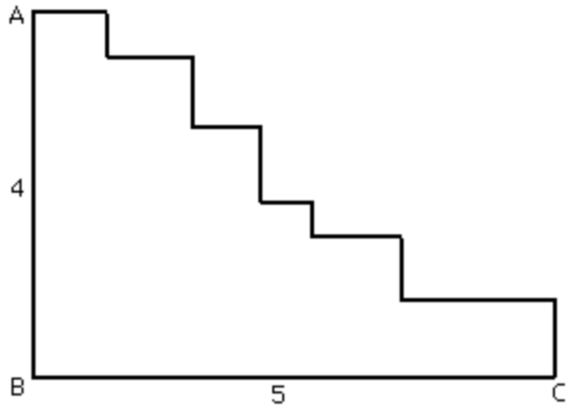
So, we can eliminate choices (D), (E) since Ron worked a little less than their friends. (A) does not seem to be fair either since Ron worked for more than 3 hours. We now need to decide between (B) and (C). ***(C)*** *seems like a good choice since it is a little less than \$9.75. This could be a good time to guess.*

What's fair?

Steve and Jerry worked for 4 hours each, while Ron worked 3 hours and 15 minutes = 3.25 h, so as a team they put in a total of 11.25 hours. Both Steve and Jerry should get $\frac{4}{11.25}$ *of \$29.25, while Ron should get a smaller fraction, namely* $\frac{3.25}{11.25} = \frac{13}{45}$ *of \$29.25 which simplifies to \$8.45. Choice* ***(C)****.*

If you do not like working with decimals, you can think of 3 hours and 15 minutes as $3\frac{1}{4} = \frac{13}{4}$ *of an hour and 4 hours as* $4 = \frac{16}{4}$ *of an hour. Altogether they worked* $\frac{13}{4} + \frac{16}{4} + \frac{16}{4} = \frac{45}{4}$ *of an hour. Thus Ron worked* $\frac{13}{45}$ *of the total time worked. Continue as above.*

71. In the diagram all the angles that are shown are right angles. The length of AB = 4 and BC = 5 as indicated in the diagram. What is the perimeter of the entire figure?



(A) 9 (B) 10 (C) 18 (D) 20 (E) 22

There are an awful lot of edges on the staircase part of this figure. However, if you imagine looking down at the staircase portion from the top, the sum of the lengths of all of the horizontal segments is 5. Similarly, if you imagine looking at the figure from the right side, the sum of the lengths of all of the vertical segments of the staircase is 4. Thus the perimeter of the whole figure is 5 + 4 + 5 + 4 = ***18****, which is choice (****C****).*

72. Name the four digit number ABCD that satisfies all of the following conditions:
i) A, B, C, D are the numbers 6, 7, 8, and 9 each used once
ii) CD forms a two digit number divisible by 4
iii) BCD forms a three digit number divisible by 3
iv) ABCD forms a four digit number divisible by 11

(A) 7876 (B) 7896 (C) 8976 (D) 9768 (E) 9876

Let's use the answers we have. ***Read*** *each condition* ***carefully*** *and* ***eliminate*** *the choices that do not satisfy them, e.g. (i) eliminates (A) since 7 is used twice and there is no 9. Other conditions require one to either remember* ***divisibility rules*** *or dividing.*
I know 80 is divisible by 4, so numbers four away from 80 or a multiple of 4 away from 80 will be also divisible by 4, so the numbers 68, 76, and 96 are all divisible by 4.
(ii) does not help. A number is divisible by 3 if the sum of its digits is divisible by 3. Checking 8+9+6 = 23 so (B) is out; 9+7+6 = 22 so (C) is out. (D) and (E) will have the same sum of 6+7+8 = 21, so let's check (iv). Either divide by 11 to check or recall the divisibility rule by 11. In the ABCD case, if (A+C)-(B+D) is divisible by 11 we are done. (9+6)-(7+8)=0 and we are done! ***(D)***

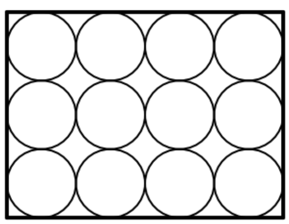
73. Every student in the 6th grade at PS 500 must read exactly three of the principal's four favorite books. If 34 read Math for Fun, 36 read Everyone Loves Math, 38 read Cool Math, and 12 read Math is for Me, how many students are in the 6th grade at PS 500?

(A) 30 (B) 40 (C) 60 (D) 90 (E) 120

Since every student reads 3 books, the total number of books read will be 3 times the number of students in the 6th grade. Thus the total number of students in the 6th grade is:

$$\frac{34+36+38+12}{3} = \frac{70+50}{3} = \frac{120}{3} = \boxed{40}$$ ***(B)***

74. Twelve round gold medals, each with a radius of three inches, just fit inside a box with four medals in each of three rows. A red tape fits snugly around the sides of this box. What is the length of the tape?



(A) 14" B) 28" (C) 42" (D) 84" (E) 422"

Feel free to ***draw on the diagram*** *and notice that a horizontal line segment through the centers of the top row would cover 8 radii. Do the same but with a vertical line and it will cover 6 radii. To wrap the box we need two horizontal and two vertical segments,* $2 \cdot 6 + 2 \cdot 8 = 28$ *radii. BUT each radius is 3" long, so the final answer is* $28 \cdot 3" = 84"$. ***(D)***

75. Two missiles are initially 5000 miles apart. They travel along a straight line directly towards one another, one traveling at 2000 miles per hour and the other at 1000 miles per hour. How many miles apart are they 1 minute before impact?

(A) 50 (B) 100 (C) 500 (D) 1000 (E) 3000

The two missiles travel a combined distance of 3000 miles in one hour, so, together, they travel 3000 miles/60 minutes = 50 miles in one minute. Therefore, one minute before impact they will be ***50*** *miles apart.* ***(A)***

Note that you did not have to figure out how far each missile traveled in one minute to solve this problem!

76. You have a 10 x 10 x 10 inch cube that is made up of little 1 x 1 x 1 inch cubes. You paint the outside of the big cube red. How many of the little cubes have paint on at least one of their sides?

(A) 456 (B) 488 (C) 598 (D) 600 (E) 1000

METHOD I: *Finding the little cubes that have* ***at least one*** *of their sides painted means we need to find the number of cubes with 1, 2, or three faces painted. Why not more? We have three different types of cubes based on their location:*

- *Corner cubes that have 3 faces painted. There are 8 of them;*
- *Edge cubes that have 2 faces painted. Since there are 12 edges and each of them will have 10-2 = 8 cubes (we need to exclude the corners since they have 3 faces painted and we already counted them), we will have* $8 \cdot 12 = 96$ *edge cubes;*
- *Face cubes that have one painted face. Since there are 6 faces and each has* $8 \cdot 8 = 64$ *that are not corner or edge cubes, we will have* $6 \cdot 64 = 384$ *cubes.*

All of them add up to $384 + 96 + 8 = 488$. ***(B)***

METHOD II: *Finding the little cubes that have* ***at least one*** *of their sides painted means we can count all of the cubes with no paint and subtract that number from the total of* $10 \cdot 10 \cdot 10 = 1,000$ *cubes.*

Inside the 10 by 10 by 10 cube we have an 8 by 8 by 8 cube as we are subtracting one from each edge. In the latter, we have $8 \cdot 8 \cdot 8 = 512$ *little cubes, so the number of cubes with no paint on is 1000 - 512 = 488.*

77. How many two digit whole numbers are increased by 18 when their digits are reversed?

(A) 6 (B) 7 (C) 9 (D) 12 (E) 14

Starting with single digits, observe that only 2 + 18 =20. Now "02" is not truly a 2-digit number, but this suggests that if we increase each digit by 1, we may get the desired result. Sure enough, 13 + 18 = 31, 24 + 18 = 42, 35 + 18 = 53, 46 + 18 = 64, 57 + 18 = 75, 68 + 18 = 86, and 79 + 18 = 97. Thus there are ***7*** *such 2-digit whole numbers (13, 24, 35, 46, 57, 68, and 79).* ***(B)***

78. When starting a new unit, a teacher announces that each day after the first, the class must do twice the total number of problems that had been assigned on all previous days. The class works for 6 days and on the 7th day she says that this is our last day of the unit. On that last day, what fraction of the problems do they still have to complete.

(A) $\frac{2}{3}$ (B) $\frac{2}{5}$ (C) $\frac{10}{31}$ (D) $\frac{1}{3}$ (E) $\frac{32}{63}$

METHOD I: *Like in #66, we are asked to compute the fraction of all the problems to be solved on the 7th day. Let's say that on the first day there is 1 problem to solve, then:*

1st day: 1

2nd day: 2(1) = 2, so on the 2nd day, students solved $\frac{2}{3}$ of the total;

3rd day: 2(1+2) = 6, so on the 3rd day, students solved $\frac{6}{1+2+6} = \frac{6}{9} = \frac{2}{3}$;

4th day: 2(1+2+6) = 18

5th day: 2(1+2+6+18) = 54

6th day: 2(1+2+6+18+54) = 162

7th day: 2(1+2+6+18+54+162) = 486

To find the fraction of the total we just need to divide:

$\frac{486}{1+2+6+18+54+162+486} = \frac{2\cdot 243}{243+2\cdot 243} = \frac{2\cdot 243}{3\cdot 243} = \frac{2}{3}$. *The answer is* ***(A).***

METHOD II: On the next day the class solves twice of the sum of problems during all previous days, so the ratio equals:

$$\frac{\textit{the last day}}{\textit{the total}} = \frac{2\cdot(\textit{the sum of problems from previous days})}{\textit{the sum of problems done from previous days} + 2\cdot(\textit{the sum of problems from previous days})} = \frac{2}{3}$$

79. How many two digit numbers are divisible by either 3 or 5?

(A) 42 (B) 46 (C) 47 (D) 48 (E) 53

Let's start by counting how many 2-digit numbers up to 99 are divisible by 3 or by 5 and then taking away the ones we don't want. Up to 99 there are 99/3 = 33 numbers that are divisible by 3 and 95/5 = 19 numbers that are divisible by 5 (95 is the largest 2-digit multiple of 5). That gives us 33 + 19 = 52 numbers. But we need to remove the 1-digit numbers (3, 6, 9, and 5) and the multiples of 15 (15, 30, 45, 60, 75, 90) since they were counted as both multiples of 3 and as multiples of 5. That leaves us with 52 - (4 + 6) = 52 - 10 = ***42*** *numbers.* ***(A)***

80. Rodney has fifty coins, including at least one quarter, that are worth \$1.00. If he loses one coin, what is the probability that it was a dime?

(A) 0.01 (B) 0.02 (C) 0.04 (D) 0.05 (E) 0.10

What coins do we know that are in use? A penny: 1¢, a nickel: 5¢, a dime: 10¢, a quarter: 25¢, a half dollar: 50¢ and a dollar coin. In this problem we do not need to consider a half dollar or a dollar coin as the values of all 50 of Rodney's coins add up to \$1. Could we have more than 1 quarter? If there are two quarters, then the remaining 48 coins would have to add up to 50¢. If we used all pennies, the greatest total of coins would only add up to 98¢. So, we can only have one quarter. The remaining 49 coins must add up to 75¢. Let's organize:

# of dimes	*# of nickels*	*# of pennies*	*total number of coins (goal 49)*
0	*0*	*75*	*75*
1	*0*	*65*	*66*
1	*1*	*60*	*62*
2	*0*	*55*	*57*
2	*1*	*50*	*53*
2	***2***	***45***	***49***
3	*0*	*45*	*48*
3	*1*	*40*	*44*

The number of coins decreases, so we found our one case with two dimes. The probability of losing a dime is then $\frac{2}{50} = \frac{1}{25} = 0.04$. ***(C)***

81. If a gym class is divided into 4 equal teams, 2 students have to sit out; 5 teams, 1 student has to sit out; 6 teams, 4 students have to sit out. If there are fewer than 70 people in the class, what is the minimum number of students that need to be added so that the class can be divided into 4, 5 and 6 equal teams with no students sitting out?

(A) 4 (B) 10 (C) 12 (D) 14 (E) 26

First observe that the number of students is even since it is 2 more than a multiple of 4 ***(all multiples of 4 are even and the sum of 2 even numbers is also even).***

Since the number is 1 more than a multiple of 5, the number ends in 1 or in 6. Since it is also even, the number ends in 6. Since the number is less than 70, it can only be 6, 16, 26, 36, 46, 56 or 66..

Since the number is neither a multiple of 4 or 6, that eliminates 6, 16, 36, 56, and 66, leaving only 26 or 46 as possibilities. However 26 has a remainder of 2 when divided by 6, so it is eliminated too. That means there were 46 students in the class.

*After 46, the next number that is divisible by 4, 5, and 6 is 60 (**the least common multiple** of 4, 5, and 6), so you need to add 60 - 46 = **14** students.* ***(D)***

82. Jana has a rectangular sheet of cardboard that measures 10 inches by 18 inches. What is the maximum number of 3-inch by 3-inch squares that she can cut from this sheet of cardboard?

(A) 15 (B) 17 (C) 18 (D) 19 (E) 20

The first instinct tells me to compute the area of the cardboard sheet, $18" \cdot 10" = 180$ *square inches, and divide it by the area of the area of one 3-inch by 3-inch square,* $3" \cdot 3" = 9$ *square units, so to look at* $180 \div 9 = 20$. *If the dimensions of the big cardboard sheet were 3” by 60” that would be the case. However, we are given a different shape and when I can fit 6 squares with a side length of 3” along the side that is 18” long, but I can only fit 3 squares along the side that is 10” long, so my true maximum is* $3 \cdot 6 = 18$ *squares. Drawing a picture helps!* ***(C)***